Fundamentals of Microwave Technology

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§ 2. Theory of Magnetron

(1) Electron Orbit

It is determined that a cathode radius is to be r_c and anode radius is to be r_a for a magnetron. The coordinates of its sectional shape is shown as Fig. 10.5, and the magnetic field B is given perpendicularly to the page toward a reader, in other words it is given in the positive z direction. Although voltage V_a is applied to the anode, the electric field by the voltage only affects a radius component. The electric field does not change components of the circumference direction and z axis direction. The dynamic equations of electrons in the space are shown as follows based on equation (6.19).

$$m\frac{d^2x}{dt^2} = eE_x + eB\frac{dy}{dt} \tag{10.1}$$

$$m\frac{d^2y}{dt^2} = eE_y - eB\frac{dx}{dt} \tag{10.2}$$

$$m\frac{d^2z}{dt^2} = 0 \tag{10.3}$$

When initial velocity is ignored, there is no motion in the z direction, and motion within the xy plane only needs to be considered. Therefore, only equations (10.1) and (10.2) are useful. When E_x, E_y respectively express electric fields in the x, y directions and V is an electric potential in any point, a following equation is given.

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y} \tag{10.4}$$

From equations (10.1), (10.2) and (10.4), a following equation is given.

$$\frac{d^2x}{dt^2}+j\frac{d^2y}{dt^2}+j\frac{eB}{m}\left(\frac{dx}{dt}+j\frac{dy}{dt}\right)+\frac{e}{m}\left(\frac{\partial V}{\partial x}+j\frac{\partial V}{\partial y}\right)=0 \quad (10.5)$$

When following equations are given, equation (10.5) can be expressed as follows.

$$z = x + jy\,,\quad z^* = x - jy \quad (10.6)$$

$$\frac{d^2z}{dt^2}+j\frac{eB}{m}\frac{dz}{dt}+\frac{2c}{m}\frac{\partial V}{\partial z^*}=0 \quad (10.7)$$

In the case the potential V is given in the following parabolic potential, equation (10.7) can be generally obtained as follows.

$$V=\frac{r^2-r_c^{\,2}}{r_a^{\,2}-r_c^{\,2}}V_a=\frac{zz^*-r_c^{\,2}}{r_a^{\,2}-r_c^{\,2}}V_a \quad (10.8)$$

From equations (10.7) and (10.8), following equations are given.

$$\sigma = r_c / r_a \quad (10.10)$$

$$\frac{d^2z}{dt^2}+j\frac{eB}{m}\frac{dz}{dt}+\frac{2e}{m}\frac{V_a}{r_a^{\,2}\left(1-\sigma^2\right)}z=0 \quad (10.9)$$

As shown in (10.9), the solution for the linear differential equation is given as follows.

$$z=\dot{R}_{1e^{a_1 t}}+\dot{R}_{2e^{a_2 t}} \quad (10.11)$$

$\dot{R}_1$ and $\dot{R}_2$ are complex numbers, and a_1 and a_2 are roots of the following equation.

$$p^2+j\frac{eB}{m}p+\frac{2e}{m}\frac{V_a}{r_a^{\,2}(1-\sigma^2)}=0$$

Namely,

$$\begin{matrix}a_1\\a_2\end{matrix}=-j\frac{eB}{2m}\left\{1\pm\sqrt{1+\frac{8m}{eB^2}\frac{V_a}{r_a^{\,2}(1-\sigma^2)}}\right\} \quad (10.12)$$

Now, a following equation is given.

$$B_c^{\ 2} = -\frac{8m}{e}\frac{V_a}{r_a^{\ 2}\left(1-\sigma^2\right)^2} \tag{10.13}$$

When the above equation is given, equation (10.12) is transformed as follows. Here, *Bc* is a critical magnetic field as explained in the following.

$$\begin{matrix} a_1 \\ a_2 \end{matrix} = -j\frac{eB}{2m}\left\{1 \pm \sqrt{1-\left(\frac{B_c}{B}\right)^2\left(1-\sigma^2\right)}\right\} = j \begin{matrix} \Omega_1 \\ \Omega_2 \end{matrix} \tag{10.14}$$

$$\Omega_1 > \Omega_2$$

Therefore, equation (10.11) is transformed as follows.

$$\begin{aligned} z &= \dot{R}_{1e^{j\Omega_1 t}} + \dot{R}_{2e^{j\Omega_2 t}} \\ &= R_{1e^{j(\Omega_1 t+\theta_1)}} + R_{2e^{j(\Omega_2 t+\theta_2)}} \\ &= \{R_{1\cos}(\Omega_1 t+\theta_1) + R_{2\cos}(\Omega_2 t+\theta_2)\} + j\{R_{1\sin}(\Omega_1 t+\theta_1) + R_{2\sin}(\Omega_2 t+\theta_2)\} \end{aligned} \tag{10.15}$$

Therefore, *x*, *y* are given as follows.

$$\begin{aligned} x &= R_{1\cos}(\Omega_1 t+\theta_1) + R_{2\cos}(\Omega_2 t+\theta_2) \\ y &= R_{1\sin}(\Omega_1 t+\theta_1) + R_{2\sin}(\Omega_2 t+\theta_2) \end{aligned} \tag{10.16}$$

From the above equations, an electron orbit can be expressed by a combined motion in which circular motions of two equal angular velocities are synthesized. When an electron starts from any point $z = r_c e^{j\theta_0}$ on the cathode plane at an initial velocity $dz/dt = 0$, a following equation can be obtained by substituting those initial conditions.

$$\begin{aligned} \dot{R}_1 &= \frac{\Omega_2}{\Omega_2 - \Omega_1} r_c e^{j\theta_0} = \frac{\Omega_2}{\Omega_1 - \Omega_2} r_c e^{j(\theta_0 - x)}, \\ \dot{R}_2 &= \frac{\Omega_1}{\Omega_1 - \Omega_2} r_c e^{j\theta_0} \end{aligned} \tag{10.17}$$

Since it is obtained $\Omega_1 > \Omega_2$ from equation (10.14), $R_2 > R_1$ is determined. Therefore, when a point C which slowly moves around the center O along the large radius *R2* at an angular velocity of Ω_2 and

a point P which moves faster around the center point C along small radius R_1 at an angular velocity of Ω_1 as shown in Fig. 10.6 are considered, the trace of point P is determined to be the electron orbit. The moderate motion of point C is called as round motion and the prompt motion around point C is called as circling motion. Depending on the magnitude of the amplitude R_1 and R_2 of the two motions, electron orbits become different.

When total energy of an electron which performs motion as in equation (10.16) is set as E, it can be given by a total of round motion energy $mR_2^2\Omega_2^2/2$, circling motion energy $mR_1^2\Omega_1^2/2$ and potential energy $\{1-(R_2^2-r_c^2)/(r_a^2-r_c^2)\}|e|V_a = \{(r_a^2-R_2^2)/(r_a^2-r_c^2)\}|e|V_a$. Therefore, when E is transformed by using equations (10.13) and (10.14), following equation can be obtained.

$$\begin{aligned} E &= \frac{m}{2}\left(R_1^2\Omega_1^2 + R_2^2\Omega_2^2\right) + \frac{r_a^2 - R_2^2}{r_a^2 - r_c^2}|e|V_a \\ &= \frac{m}{2}R_1^2\Omega_1^2 + \frac{m}{2}R_2^2\left\{\Omega_2^2 - \left(\frac{eB}{2m}\right)^2\left(\frac{B_c}{B}\right)^2\left(1-\sigma^2\right)\right\} + \frac{|e|V_a}{1-\sigma^2} \\ &= \frac{m}{2}R_1^2\Omega_1^2 - \frac{m}{2}R_2^2\Omega_2^2(\Omega_1 - \Omega_2) + \frac{|e|V_a}{1-\sigma^2} \end{aligned} \qquad (10.18)$$

Here, it should be noted that the second term is negative since $\Omega_1 > \Omega_2$. Therefore, the total energy decreases/increases when R_1 is constant and R_2 increases/decreases. In other words, although round motion energy increase when the amplitude of round motion R_2 increases, potential energy deceases more than the increase of round motion energy. This is crucially important in considering oscillation mechanism of a magnetron.

(2) Oscillation Mechanism

It has been understood that electrons take an orbit which is synthesized by two periodic motions at angular velocity Ω_1, Ω_2 in the case the distribution of electrostatic potential within a magnetron is given by a parabolic potential. However, in order the periodic motion of electrons to induce electric oscillation to an external circuit, the electrons needs to make a movement with a mass of space electric charges by aligning the phase of each electron to some extent. Even if respective electrons make round movements, effects for an external circuit are reversed when respective electrons freely make movements in different phases, thereby electric oscillation is not induced. In other words, phase focusing of electrons is required. In the case the phase-focused motion electrons promotes oscillation of an external circuit by supplying energy to the external circuit, the oscillation continues while compensating the loss. The factor for focusing the phase of electrons is the electric oscillation occurred by random impact to the external circuit.

The electric oscillation occurred in the external circuit is generally weak. However, the amplitude R_1 or R_2 infinitely increases by resonance in the case an angular velocity of ω of the oscillation corresponds to Ω_1 or Ω_2. Therefore, by equation (10.16) electrons initially located nearby the cathode gradually spread to the radius direction to finally reach the anode.

It should be noted that the motion of the angular frequency of Ω_2 has negative energy and the motion of the angular frequency of Ω_1 has positive energy. When $\omega = \Omega_1$, the amplitude increases by oscillation, and energy increases by that amount. In other words,

oscillation in the circuit is decreased since energy in the external circuit is used to promote electron motion of the angular frequency Ω_1. On the other hand, when $\omega = \Omega_2$, the amplitude R_2 increased by oscillation and the absolute value of energy is increased by that amount. However, the energy is given to the external circuit to promote oscillation of the circuit. In conclusion, the angular frequency ω of oscillation in the circuit needs to correspond to Ω_2 for inducing oscillation in the external circuit, and oscillation in the circuit disappears in the case ω is equal to Ω_1.

In the following, how the phase of electrons is focused is considered by a segmented anode magnetron shown in Fig. 10.7. Since equipotential lines form concentric circles when there is no oscillation in the external circuit, the electrons make a movement along the equipotential lines and amplitude R_2 does not change. However, when there is oscillation in the external circuit as shown in Fig. 10.7, phase focusing of electrons is different. In other words, in the case an anode segment A has decreased voltage than B does, the equipotential lines do not form concentric circles and protrude to the decreased voltage side A as shown in Fig. 10.17. At this moment, electrons passing a gap 1 of both anodes are pulled toward outside, and electrons passing a gap 2 are pushed inside on the other hand. Therefore, amplitude R_2 of the electrons passing the gap 1 increases whereas amplitude R_2 of the electrons passing the gap 2 decreases. Phase of the external electric oscillation changes while electrons whose R_2 is increased at the gap 1 reach the gap 2. However, since the equipotential lines are approximately in a circular arc in positions other than the gaps, R_2 does not change. When $\omega = \Omega_2$ is established,

the electric oscillation has passed a half cycle by the time electrons reach the gap 2, and the anode segment B has decreased voltage than A conversely. Therefore, R_2 increases again at the gap 2, and electrons initially positioned at the gap 2 again decrease R_2 at the gap 1. The former is a so-called good electron which contributes to oscillation, and the latter is a bad electron which does not contribute to oscillation.

Here, it is considered a moment in which electric oscillation in the external circuit is at the maximum. When the anode segment A has decreased (increased) voltage than B does, electrons positioning the right (left) half increase amplitude R_2 every time passing the gap, whereas electrons positioning the left (right) half decrease amplitude R_2 every time passing the gap. Therefore, at the time when oscillation is at the maximum, electrons positioning the right (left) half are good electrons and electrons positioning the left (right) half are bad electrons. However, the bad electrons do not remain to be bad electrons to decrease amplitude R_2, but increase amplitude R_2 by joining good electrons eventually. In the manner as described above, in the case $\omega = \Omega_2$ is established, the phase of electrons gradually aligns by always collecting the electrons at the right (left) gap at the moment when oscillation is at the maximum and the anode segment A has the decreased (increased) voltage than B does. The collection of electrons goes around within a conduit at an angular velocity of Ω_2. Such collection of electrons is referred to as a rotating electron pole.

Fig. 10.8 shows a state of rotation of a rotating electron pole. Since the electron pole has negative charge, an electrode to which the electron pole approximates generates positive charge by electrostatic induction whereas an electrode from which to electron pole recedes

generates negative charge. As apparent from Fig. 10.8, positive charge is induced when electric potential of the anode is increased whereas negative charge is induced when electric potential of the anode is decreased. Thus, amplitude of electric oscillation is increased by the rotation of an electron pole to promote oscillation.

(3) B-Type Oscillation (Oscillation by Traveling Wave Type)

Here, oscillation conditions of a multifraction anode magnetron will be obtained. A multifraction anode magnetron (division number N) is shown in Fig. 10.9. There are phase differences between high-frequency voltage of one anode gap and high-frequency voltage of the following anode gap to form a rotary alternating electric field in an interaction space between a cathode and anode. When phase difference between adjacent segments is set as φ, there are N_φ of phase transitions by making a circle. In the case of a cylindrical anode, the start point and end point are end up with the same segment. Therefore, the phase transitions N_φ making a circle must be an integral multiple of 2π. Namely, an equation shown in the following is obtained.

$$\varphi = \frac{2\pi\, n}{N}, \quad n=0,\ 1,\ 2,\ \ldots,\ N/2 \qquad (10.19)$$

$\varphi = \pi$ is satisfied when $n=N/2$, and the oscillation at this time is called as π mode. Fig. 10.9 shows a case of π mode.

Since a rotary angular velocity of an electron is Ω_2, it is required the time of $2\pi/\Omega_2$ for making a circle in the interaction space. The time τ which is required for an electron to pass adjacent gap between anodes is obtained by the following equation.

$$\tau = \frac{2\pi}{N\Omega_2} \tag{10.20}$$

When the rotary direction of electrons and the rotary direction of an electric field are the same, the phase difference by which an electron experiences at a gap between adjacent anodes is expressed by $\omega\tau - \varphi$. Therefore, when the phase difference corresponds to an integral multiple of 2π, electrons encounter in the same phase at each gap between anodes. In consequence, following equations are obtained.

$$\omega\tau - \varphi = 2\pi\, p, \quad p = 0, \pm 1, \pm 2\ldots \tag{10.21}$$

From equations (10.19), (10.20) and (10.21), following equations are obtained.

$$\omega = \left| \left(p + \frac{n}{N} \right) N \right| \Omega_2 = |k| \Omega_2 \tag{10.22}$$

$$k = \left(p + \frac{n}{N} \right) N \tag{10.23}$$

Equation (10.22) shows the relationship between the oscillating frequency and rotary angular velocity of an electron. k may be any value from equation (10.23), however the oscillation mode is different depending on its value. The most common mode is π mode of $\varphi = \pi$, and k can be obtained from the following equation.

$$k = \left(p + \frac{1}{2} \right) N \tag{10.24}$$

Fig. 10.10 shows electric potential of respective anode segments in propagation mode in which phase transitions between adjacent segments are equal to π. The electric potential distribution is shown in time intervals of $T/2$, namely every half cycle. The figure shows a linear view of 8 divisional anodes.

In the case electron velocity and rotation velocity of the electric field are equal, in the case p=0, k=4 is obtained from equation (10.24). In other words, electrons pass through a course shown in the figure and high frequency waves change by a half cycle while the electrons pass between adjacent segments. The interaction with the electric field is closest in this case. In the case electron velocity is slower than rotation velocity of the electric field, for example p=1 and k=12, high frequency waves change by one and a half cycles while the electrons pass between adjacent segments. When p=2 and k=20, the result becomes as shown in Fig. 10.10. There is also a probability of energy delivery when electron velocity is faster than rotation velocity of the electric field. That is when p=0 at π mode in the case of divided anode referred in the previous section.

(4) Operation Line

From Fig. 10.6, when the maximum radius $r_{\max}$ of the electron orbit is equal to a radius r_a of an anode, following equations are satisfied.

$$\begin{aligned} r_{\max} &= r_a = R_1 + R_2 \\ &= \frac{\Omega_1 + \Omega_2}{\Omega_1 - \Omega_2} r_c \qquad\qquad (10.25) \\ &= \frac{r_c}{\sqrt{1 - \dfrac{8m}{|e|B^2} \dfrac{V_a}{r_a^{\,2}\left(1-\sigma^2\right)}}} \end{aligned}$$

The electrons do not reach the anode in the case they are not less than B in the above equation. This is the afore-mentioned critical magnetic field, and the following equation is obtained when it is set as B_c.

$$B_c = \frac{8m}{|e|} \frac{V_a}{r_a^{\,2}\left(1-\sigma^2\right)^2}$$

Therefore, the magnetic field given by equation (10.13) expresses the critical magnetic field. When r_a is expressed in cm, V_a is in volt and B_c is in gauss, the following equation can be obtained and the relationship between B_c and V_a is to be a parabola.

$$B_c = \frac{6.72}{r_a\left(1-\sigma^2\right)}\sqrt{V_a} \qquad (10.26)$$

The oscillation condition of a divided anode magnetron is given by equation (10.22). Since the case of p=0 is generally used, the following equation can be obtained when $\omega = n\Omega_2$.

$$\frac{\omega}{n} = \frac{|e|B}{2m}\left\{1-\sqrt{1-\frac{8m}{|e|B^2}\frac{V_a}{r_a^{\,2}\left(1-\sigma^2\right)}}\right\}$$

When λ is set as wavelengths, the following equation can be obtained for V_a.

$$V_a = \frac{\pi\, c r_a^{\,2}\left(1-\sigma^2\right)}{n\lambda}\left(B-\frac{2\pi\, cm}{|e| n\lambda}\right) \qquad (10.27)$$

When r_a and λ are expressed in cm and B is in gauss, V_a expressed in volt is given as the following equation.

Substitute equation (10.10)

$$V_a = \frac{942 r_a^{\,2}\left(1-\sigma^2\right)}{n\lambda}\left(B-\frac{10650}{n\lambda}\right) \qquad (10.28)$$

An operation line in the case electric potential distribution is proportional to a square of the radius as in equation (10.8) is given by equation (10.28).

The same result can be obtained from the relationship between energy and work. In other words, in circular coordinates r, O,

a dynamic equation can be obtained as follows when F_r, F_θ are set as components for orthodromic force of r, O.

$$m\left\{\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right\} = F_r, \quad \frac{d}{dt}\left(mr^2\frac{d\theta}{dt}\right) = rF_\theta \qquad (10.29)$$

A relational expression of work and energy can be obtained by multiplying dr/dt and the above first equation and by multiplying $d\theta/dt$ and the above second equation.

$$\frac{d}{dt}\left\{\frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right]\right\} = F_r\frac{dr}{dt} + F_{\theta r}\frac{d\theta}{dt} \qquad (10.30)$$

Components of force F change as time in a coordinate system at rest. However, the components are independent to time in a system rotating with the same velocity as high-frequency electric field. Now, equation (10.30) is transformed to an equation in a rotary coordinate system. When the coordinate system is rotating at an angular velocity Ω_2, $\Omega_2 = \omega/n$ is obtained in the case of operation under n mode. Since n is the number of wavelengths nearby the anode, high-frequency electric field, namely the coordinate system proceeds only by $2\pi/n$ during a high-frequency period T, and the angular velocity Ω_2 is to be $\Omega_2 = \frac{2\pi}{n}\frac{1}{T} = \omega/n$. A rotary coordinate system being $\theta' = \theta - \Omega_2 t$ is used for equation (10.29) and dr/dt and $d\theta'/dt$ are added by respectively multiplying, the following equation can be obtained.

$$\frac{d}{dt}\left\{\frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta'}{dt}\right)^2\right]\right\} = F_r\frac{dr}{dt} + F_{\theta r}\frac{d\theta'}{dt} + \frac{d}{dt}\left(\frac{1}{2}m\Omega_2{}^2 r^2\right) \qquad (10.31)$$

The above-described equation is a relational expression between work and energy in a rotary coordinate system. Effects of the magnetic field are now considered in the case the magnetic filed B is present only in

the z axis direction. In this case, components of force are given by equations, $F_r = eB_r \frac{d\theta}{dt}$, $F_\theta = -eB\frac{dr}{dt}$ from equation (6.20). When those equations are substituted for equation (10.31), the right-hand side of the equation is given as follows.

$$\frac{d}{dt}\left(\frac{1}{2}m\Omega_2{}^2 r^2 + \frac{1}{2}eB\Omega_2 r^2\right) = \frac{d}{dt}\left[\frac{1}{2}\left(m\Omega_2{}^2 + eB\Omega_2\right)r^2\right]$$

The second equation of equation (10.29) is shown as follows in a rotary coordinate system.

$$\frac{d}{dt}\left(mr^2\frac{d\theta'}{dt}\right) = \frac{d}{dt}\left(-\frac{eBr^2}{2} - m\Omega_2 r^2\right) \qquad (10.32)$$

The case when an electric field is given by overlapping electrons and a magnetic field is now considered. The electric field includes a direct electric field, applied high-frequency electric field and high-frequency electric field generated by space charge. However, the composite electric field is indifferent to time in a rotary coordinate system. In the case the force by the electric is defined as F', the relational expression (10.31) between work and energy and equation (10.32) are expressed as follows.

$$\frac{d}{dt}\left\{\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta'}{dt}\right)^2\right]\right\} = F_{r'}\frac{dr}{dt} + F_{\theta' r}\frac{d\theta'}{dt} + \frac{d}{dt}\left[\frac{1}{2}\left(m\Omega_2{}^2 + eB\Omega_2\right) r^2\right] \qquad (10.33)$$

$$\frac{d}{dt}\left(mr^2\frac{d\theta'}{dt}\right) = rF_{\theta'} + \frac{d}{dt}\left[\left(-\frac{eB}{2} - m\Omega_2\right)r^2\right] \qquad (10.34)$$

In the case of a magnetron, $F_{\theta'} = 0$ is satisfied. The critical point is when electrons jump out from the cathode surface at zero speed and jump in the anode surface of $r = r_a$ at $dr/dt = 0$ as mentioned above.

In such a case, following equation can be obviously obtained by using a coordinate system at rest with $\Omega_2 = 0$ from equation (10.34).

$$mr^2 \frac{d\theta}{dt} = \frac{-eB}{2}\left(r^2 - r_c^{\ 2}\right) \tag{10.35}$$

Moreover, following equation can be obtained by integrating equation (10.33) with respect to time and using the above equation.

$$\frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^3 + r^2\left(\frac{eB}{2m}\right)^2\left(1-\frac{r_c^{\ 2}}{r^2}\right)^2\right] = \int_{r_c}^{r} F_r dr$$

From the above equation, $dr/dt = 0$ is given when $r = r_a$ and when voltage of the anode is V_a, following equation which corresponds to equation of the critical magnetic field in (10.13) can be obtained.

$$-eV_a = \frac{m}{8}\left(\frac{eB}{m}\right)^2 r_a^{\ 2}\left(1-\frac{r_c^{\ 2}}{r_a^{\ 2}}\right)^2 \tag{10.36}$$

An equation which gives an operation line can be obtained in the following manner. As mentioned above, suppose that electrons jump out at zero speed in the cathode and reach the anode at an extreme small value of dr/dt and $d\theta'/dt = 0$, the following equation can be obtained from equation (10.33).

$$\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta'}{dt}\right)^2\right] = \int_{r_c}^{r} F_{r'}dr + \frac{m}{2}\Omega_2^{\ 2}r^2 + \frac{1}{2}eB\Omega_2\left(r^2 - r_c^{\ 2}\right)$$

Since the right-hand side of the equation is 0 when $r = r_a$, following equation can be obtained by using $\Omega_2 = \omega/n = 2\pi\, c/(n\lambda)$.

$$\begin{aligned} V_a &= \frac{1}{2}B\Omega_2\left(r_a^{\ 2} - r_c^{\ 2}\right) + \frac{m}{2e}\Omega_2^{\ 2}r_a^{\ 2} \\ &= \frac{\pi\, c}{n\lambda} r_a^{\ 2}\left(1-\sigma^2\right)\left\{B - \frac{m}{|e|}\frac{2\pi\, c}{n\lambda}\frac{1}{1-\sigma^2}\right\} \\ &= \frac{942 r_a^{\ 2}\left(1-\sigma^2\right)}{n\lambda}\left(B - \frac{10650}{n\lambda\left(1-\sigma^2\right)}\right) \end{aligned} \tag{10.37}$$

The equation is called as Hartree line and is slightly different from the equation (10.28). Fig. 10.11 shows a critical curve expressed by equation (10.26) or (10.36), operation line obtained from equation (10.28) and Hartree line obtained from equation (10.37). In the figure, the horizontal axis shows magnetic field intensity and the vertical axis shows anode voltage. The critical curve is shown as a parabola and intersects with the operation line at two points. The critical curve and the Hartree line intersect at a point B_{OH}, and its value is obtained by the following equation. The value is called as marginal magnetic field intensity.

$$B_{0H} = \frac{2m}{|e|}\frac{\Omega_2}{\left(1-\sigma^2\right)} = \frac{21400}{n\lambda\ (cm)\left(1-\sigma^2\right)} gauss \qquad (10.38)$$

The voltage V_{OH} corresponding to the marginal magnetic field intensity is called as marginal anode voltage and can be obtained form the following equation.

$$V_{0H} = \frac{1}{2}\frac{m}{|e|}\Omega_2{}^2 r_a{}^2 = 10.08\times10^6 \frac{r_a{}^2}{n^2\lambda^2} volt \qquad (10.39)$$

The equations of the critical magnetic field and Hartree line can be expressed by the terms in the above equation.

$$\text{Critical magnetic field: } \frac{V_a}{V_{0H}} = \left(\frac{B}{B_{0H}}\right)^2 \qquad (10.40)$$

$$\text{Hartree line: } \frac{V_a}{V_{0H}} = 2\frac{B}{B_{oH}} - 1 \qquad (10.41)$$

The operation line and critical curve intersect at two points as mentioned above. When the magnetic field of the right side of intersecting points is set as B_O, the following equation can be obtained.

$$B_0 = \frac{4\pi\ mc}{|e| n\lambda\ (1-\sigma)} = \frac{21400}{n\lambda\ (1-\sigma)} gauss \tag{10.42}$$

There is no oscillation in the magnetic field smaller than B_0, since electrons are taken by the anode.

When round velocity of electrons and that of high-frequency electric field are synchronized, electrons give energy to the circuit. Therefore, the radius R_2 enlarges and the electrons are taken by the anode. When the energy of motion of the electrons is ignored since velocity to the radius direction is smaller than velocity to the circumferential direction, the following equation can be obtained.

$$\frac{m}{2}\{(r_a - R_1)\Omega_2 + R_1\Omega_1\}^2 = \frac{m}{2} r_a{}^2 (1+\sigma)^2 \Omega_2{}^2$$

Electrons have the potential energy of $|e|V_a$ when they start the surface of the cathode. Therefore, electron efficiency η_e is defined and calculated as follows.

$$\eta_e = \frac{|e|V_a - \frac{m}{2} r_a{}^2 (1+\sigma)^2 \Omega_2{}^2}{|e|V_a} = 1 - \frac{1+\sigma}{\frac{2B}{B_0} - 1 + \sigma} \tag{10.43}$$

The relationship between η_e and B is also shown in Fig. 10.11. Moreover, the optimum radius ratio is given by the following equation.

$$\sigma = \frac{r_c}{r_a} = 0.85 - \frac{3.83}{N} \tag{10.44}$$

§ 3. Resonance Circuit of Magnetron

(1) Various Types of Resonance Circuits

Various types of resonance circuits are used for magnetrons, and magnetrons may be classified based on resonance circuits.

However, resonance circuits shown in the following have been generally used nowadays. Namely, following types of circuits are used.

(a) slot type

(b) hole and slot type

(c) vane type

(d) rising-sun type

Fig. 10.12 shows shapes of anodes. There are various division numbers of resonance circuits and the number of which is determined based on wavelengths and magnetic flux density or a value of anode voltage. In the case division number of anodes is identical as shown in equation (10.22), rotary angular velocity of electrons needs to be faster to obtain oscillation in high frequency. To increase the rotary angular velocity of electrons, however, higher magnetic field and anode voltage are required from equation (10.14) and this is disadvantage in practical applications. Therefore, division number N needs to be larger in order to mitigate this problem.

In the types shown in Figs. 10.12(a), (b) and (c), respective resonance circuits are not mutually connected. Therefore, each resonance frequency is supposed to be identical with entire resonance frequency when resonance frequencies of respective resonance circuits are all identical. However, entire resonance frequency is separated into various different values since electromagnetic coupling or capacitive coupling is present between respective resonance circuits. Furthermore, since phase difference between adjacent divided segments changes depending on the number of resonance modes as shown in equation (10.19), resonance frequency changes by the changes of coupling conditions. In the case the number of oscillation

modes changes, oscillation conditions would be different. However, operation becomes extremely unstable by generating significant leap of oscillation modes through slight changes of an operation point, in the case operation conditions of adjacent modes do not change much. It has been known that π mode is efficient, therefore, π mode needs to be separated from other modes. This is referred to as π-mode separation. In order to perform the separation, anode segments are connected with straps of a conductor to oscillate a number of resonance circuits in a fixed phase relation as shown in Fig. 10.12(f). In other words, since two straps are respectively connecting every other anode segments, electric potential of every other anode segments is forced to oscillate at the identical phase. Alternatively, anodes with the rising sun type as shown in Figs. 10.12(d) and (e) may be used.

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